Limits to Growth Concepts in Classical Economics

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Abstract

Neoclassical economics seems to have ignored the concept of physical limits to growth by assuming that the market and the technological advances invoked by it will make it possible to tap new resources and create substitution of production factors, while it has outright excluded limitations invoked by the political, psychological and social institutions in its analyses. Classical economics, on the other hand, appears to have been cognizant of a multitude of limitations to growth, including demographic, environmental, and social. In this paper, I reconstruct classical economic growth models using system dynamics method and explain their behavior using computer simulation. The paper not only demonstrates that system dynamics can be used with advantage for constructing models of theoretical concepts in economics and experimenting with them, it also makes a case for taking a pluralistic view of the growth process and reincorporating a multitude of institutions driving it into our models to arrive at realistic policy options.

Key words: economic growth, economic development, economics, classical economics, system dynamics, computer simulation, environment, limits to growth.

Introduction

This paper reconstructs the demographic, environmental, and social limits to growth as posited in the classical economic growth models of Adam Smith, David Ricardo, Thomas Malthus, Karl Marx and Joseph Schumpeter. System dynamics modeling and computer simulation are used to demonstrate the systemic perspective and the richness of these models. The multiplicity of the institutions and the non-quantifiable factors the classical economics models took into account while attempting to explain the dynamics of the growth process, according to Baumol (1959), indeed described magnificent dynamics that were relevant to their respective empirical contexts. The purpose of the paper is to provide a vehicle for understanding classical thought on economic growth and to reiterate the importance of the variety of behavioral and demographic factors and the non-quantifiable soft variables it subsumed. In the complex world of today, it would be impossible to ignore these variables without losing sight of the important dynamics that we experience in reality. As an original content analysis of the classical writings is not intended, where possible the models of this paper draw form secondary interpretations. In particular, mathematical formulations of classical theories by Higgins (1968) provided the inspiration as well as the basic structure of the system dynamics models I present in the paper.

The concept of limits in economics

Neoclassical economics mostly excluded environmental, demographic and social limitations from its formal analyses until early 1970s, although it extensively addressed the periodic limitations to growth arising out of the stagnation caused by imbalances in the market. As an exception, Hotelling (1931) dealt with exhaustible resources with concerns that the market may not be able to return optimal rates of exhaustion, but without pessimism about the technology to bring to fore new sources as old ones are exhausted. These early concerns have been followed by a blissful confidence in the ability of the technological developments and prices to provide access to unlimited supplies of resources (Devarajan and Fisher 1981, Smith and Krutilla 1984).

Solow's 1974 Richard T Ely lecture made a strong argument for integrating depletion of resources into the models of economic growth (Solow 1974), but the bulk of work in orthodox economics has nonetheless not deviated much from its earlier focus on optimal rates of depletion and pricing of resources (Nordhaus 1964, 1979) without concerns for environmental capacity, which are mostly expressed in passing. There have been some concerns also expressed about intergenerational equity, but its treatments remain tied to arbitrary rates of discount (Hartwick 1977, Solow 1986). Environmental analysis seems to have appeared as an add-on in response to the environmental movement spearheaded by the famous *Limits to Growth* study (Forrester 1971, Meadows, et. al. 1972, 1974, 1992). In this add on, the neoclassical economic theory has continued to assume mineral resources to be unlimited and to expect prices and technological developments to continue to unearth richer mines so existing mines may be abandoned (Saeed 1985). The reality of political power, the creation and resolution of social conflict and the psychological and behavioral factors also remain excluded from the classical analysis, although they contribute significantly to the performance of the economies (Street 1983).

Classical economics, on the other hand seems to have addressed a rich variety of limiting factors covering social, political, demographic and environmental domains, often dealing with soft variables that are difficult to quantify but that have significant impact on behavior of the economy. In particular, the growth models proposed by Adam Smith, Karl Marx, David Ricardo, Thomas Malthus and Joseph Schumpeter dealt with such limiting factors that have often been ignored in the mathematical tradition of neoclassical economics, although these can be easily incorporated into our models using system dynamics.

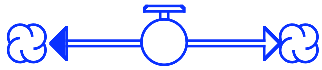
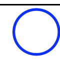
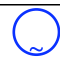
System dynamics modeling

System dynamics modeling, originally introduced by Jay Forrester in the1950s to address problems of industrial management (Forrester 1961), came in limelight with the publication of the controversial *Limits to Growth* study in 1972 (Meadows, et. al. 1972). Although this association extended to some degree the ambivalence experienced by the Limits study also to system dynamics, the methodology is in fact quite neutral and invaluable for exploring the behavior of a given set of structural assumptions (Forrester 1979). It also allows us to subsume both quantifiable and non-quantifiable variables into formal models and understand the structure of the classical growth theories with relative ease, which is attempted in this paper. Barry Richmond's little known attempt to model a slice of the ideas expressed by Adam Smith and David Ricardo, based on an interpretation by Heilbroner (1980), demonstrates the richness of the classical thought and how system dynamics modeling can capture and communicate it in an intuitive language (High Performance Systems 1997). Other notable examples of use of system dynamics to construct realistic theories addressing economic policy include Forrester (1973), Mass (1975), and Saeed (1994).

The underlying computational process in a model representing the microstructure of a theory can be expressed as a set of ordinary nonlinear integral equations. System dynamics modeling allows us to construct such models using icons and connections that can be easily assembled using dedicated software like ithink and Vensim.[1] Table 1 shows the icons and the processes they represent in a typical system dynamics model. A rectangle represents a stock that integrates the flows connected to it. A flow is a rate of change associated with a stock which may have more than one flows connected to it. These two types of variables are the basic components of an abstract system implicit in a theory. Information links from stocks to flows define decision rules. Intermediate computations transforming information in stocks into decision rules are represented by the converter symbol. A converter is an algebraic function of stocks, other converters and constant parameters. When an intermediate computation involves a nonlinear graphical relationship between two variables, a tilde is added to the converter symbol.

[1] ithink is a trademark of isee systems, Inc.; Vensim is a trademark of Ventana Systems, Inc.

Table 1 Icons used for representing model relationships

Process	Icon	Explanation
Stock		Accumulation or integration of flows linked to the icon
Flow		A rate of change or a derivative of a stock. Empty arrowhead indicates normal direction of flow. Normally connected to a stock. Cloud at one end represents unlimited source or sink
Converter		Algebraic function of stocks, other converters and constants
Graphical function		Graphically represented function of another variable in the system
Causal link		Information relationship between two variables

Once a model has been constructed using these icons, the software allows the modeler to specify initial values of stocks, constant parameter, algebraic functions and graphical relationships that define each stock, flow and converter. Non-quantifiable variables are often represented by indices that are normalized with respect to a given ambient condition. The software also allows us to generate the behavior of these models through numerical simulation on a computer. The simulation experiments are often designed to understand the behavioral patterns arising out of the model structure rather than point prediction of the future.

The models of this paper are represented graphically and programmed in *ithink* software using the icons in Table 1. The mathematical equations for each model are summarized in tables and text accompanying graphical representation of each model discussed. Computer simulations of these models generate the dynamic behavior postulated by the respective classical theories. Machine-readable versions of these models are available from the author on request.

Adam Smith and the implicit demographic limit to growth

Although Adam Smith did not clearly discuss the limits to growth, a demographic constraint is implicit in his model since labor is an autonomous production factor assumed to be freely available, while capital and technology are endogenously created through investment of profits (Smith 1977). Also, land which is a proxy for renewable resources, can be freely substituted by capital (Higgins 1968, pp 56-63), hence it can be aggregated with capital. Figure 1 represents in system dynamics terms the relationships between the production factors and the output postulated by Adam Smith.

At the outset, output is created by capital, labor and technology. A labor constraint on output appears when capital-labor ratio is suboptimal. Capital increases through investment, which is driven by profits determined by the difference between output and wage bill. Technological growth is also driven by investment meaning that new capital formation will upgrade technology. Labor can be hired form a pool of unemployed that is fed by population growth, while wage depends on the tightness of the labor market.

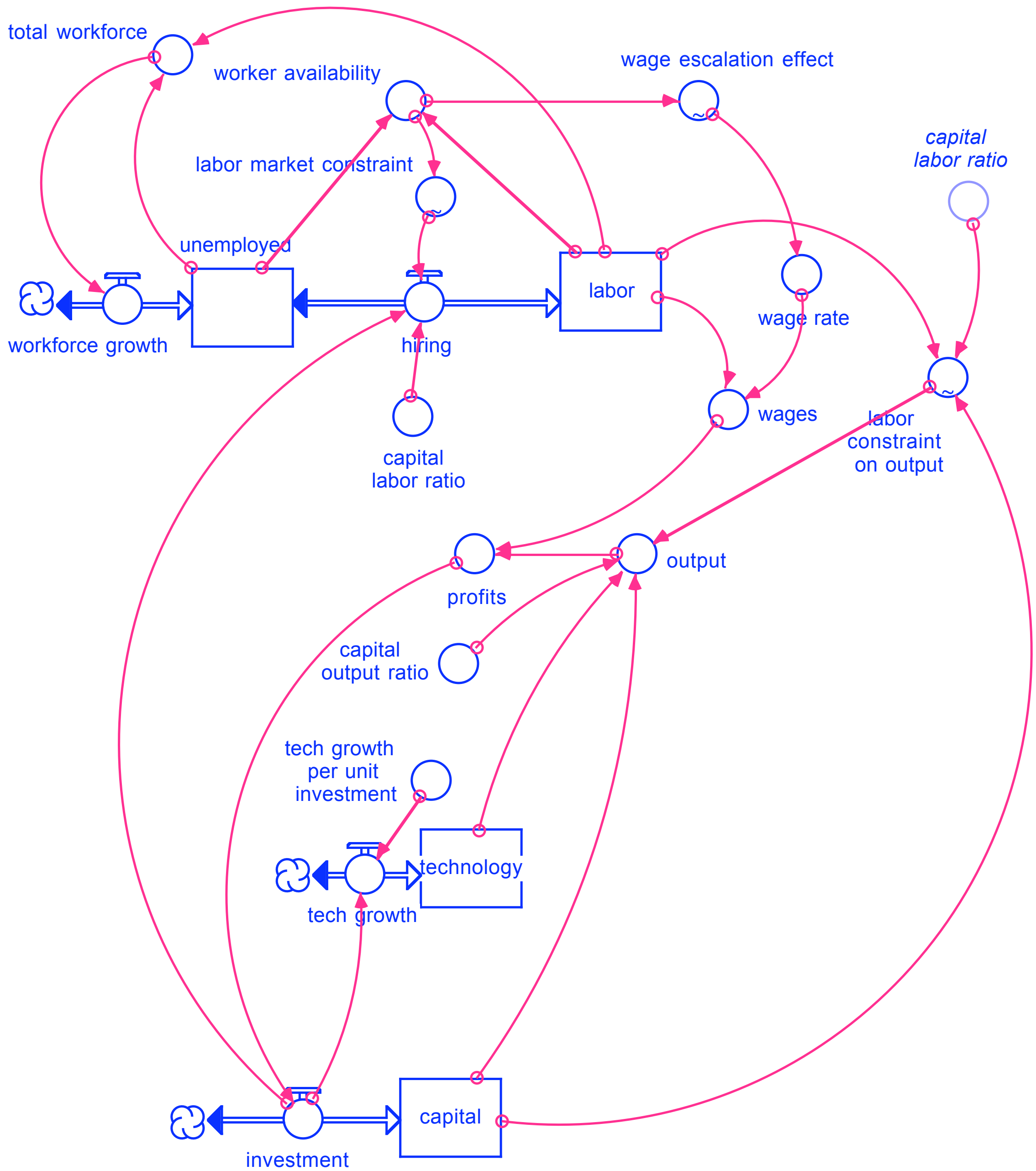


Figure 1 Growth of output and production factors in Adam Smith's model

Table 2 Mathematical relationships in the model of Figure 1

VARIABLES	COMPUTATIONAL EQUIVALENTS
a) Stocks and Flows	
Capital	∫(investment)dt
investment	profits
Labor	∫(hiring)dt
hiring	(investment/capital labor ratio)*labor market constraint
unemployed	∫(workforce growth- hiring)dt
workforce growth	total workforce*fractional workforce growth rate
technology	∫(tech growth)dt
tech growth	investment*tech growth per unit investment
b) Converters	
output	technology*(capital/capital output ratio)*labor constraint on output
Profits	output-wages
total workforce	labor + unemployed
wages	labor*wage rate
wage rate	normal wage*wage escalation effect
worker availability	(unemployed/labor)/(INIT unemployed/INIT labor)
c) Graphical functions	
labor market constraint	f_1(worker availability); $f_1' > 0$, $f_1'' < 0$
labor constraint on output	f_2(labor/(capital/capital labor ratio)); $f_2' > 0$, $f_2'' < 0$
wage escalation effect	f_3(worker availability); $f_3' < 0$; $f_3'' < 0$
d) Initial values of stocks	
INIT capital	100
INIT labor	10
INIT unemployed	2
INIT technology	1
e) Constant parameters	
capital labor ratio	10
capital output ratio	1
tech growth per unit investment	.0004
normal wage	8
fractional workforce growth rate	0

Table 2 gives computations underlying each icon in the model of Figure 1. Since a numerical simulation process is used, the model must be supplied with initial values of stocks and constant parameters even though we might only be interested in qualitative patterns of behavior. These values are selected for internal consistency so a hypothetical homeostasis exists while output exceeds the wage bill thus returning a positive value of profits.

If a labor market constraint did not exist, this system would create three powerful positive feedback loops driven by growth in capital, labor and technology, which would cause explosive growth as long as the wage bill remains less than the output and the system yields positive profits. However, when labor is hired from a fixed pool of unemployed, the labor market becomes tight and wage rate escalates. This drives profits to zero pretty quickly. Thus, in the absence of growth in population (total workforce in the model), the system equilibrates at full employment as shown in the simulation of Figure 2.

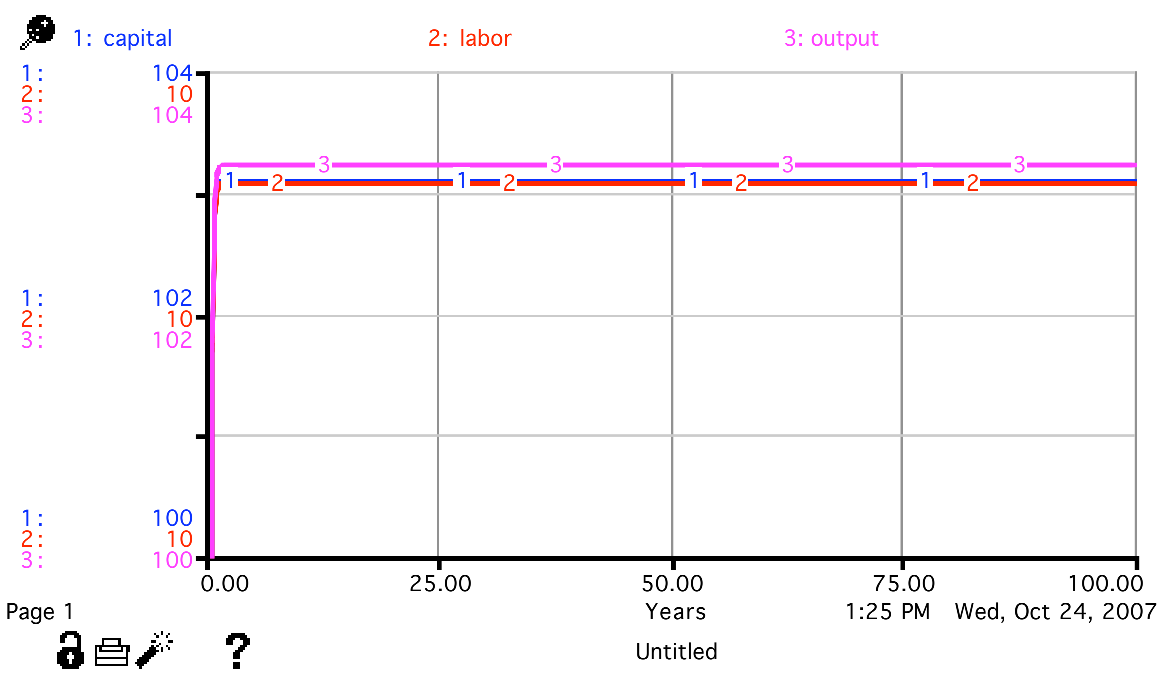


Figure 2 Behavior of the model with demographic constraints

When land is aggregated with capital in the production schedule, the capital input and the labor constraint return a Cobb Douglas type production function in which the influence of technology is autonomous. Growth in any one of the inputs to production can create a growth in output, however, while Adam Smith gave an endogenous explanation of how capital and technology grew, he did not discuss any limitations on the growth of labor, assuming in default that population growth would continue to provide sufficient quantities of labor so the labor constraint on output does not become active and wage escalation does not occur. Investment, which is driven by profits, drives all: capital formation, technological growth and labor hiring.

A sustained growth in this system is possible only when a growth in the total workforce can sustain a pool of unemployed that also keeps wage rate from escalating. Indeed, a sustained growth is obtained when the model of Figure 1 is simulated with a 2% workforce growth rate (fractional workforce growth rate is changed to .02). This is shown in the simulation of Figure 3. Clearly, population growth that creates a growing supply of labor is critical to maintaining economic growth in Adam Smith's model. Hence, the demographic constraint is the unwritten limit to growth since all else is driven by the profits, which would decline to zero when a tight labor market caused by a fixed population creates wage escalation.

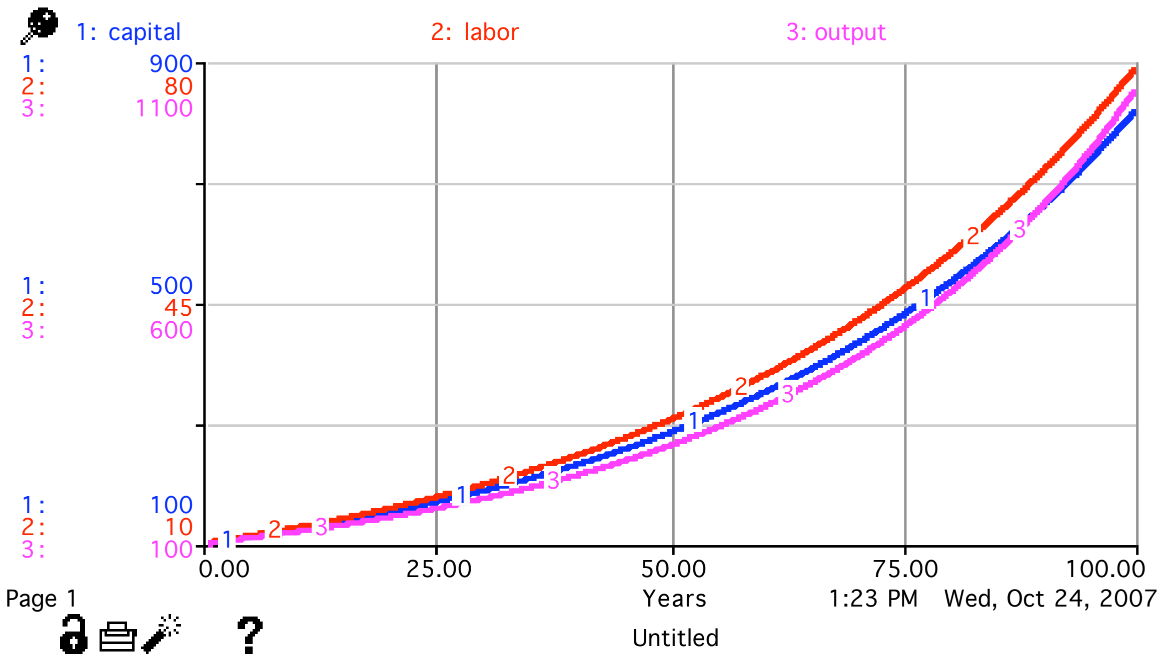


Figure 3 Economic growth supported by population growth

It should also be noted that there is no surplus or deficit of supply and demand in the model and all production is consumed, implicitly meaning that both profit and wage components are distributed to the households, hence the demand for goods and services depends on the total income rather than a part of it. This implies that capital ownership is widespread, which creates household claims to profit across board. This assumption seems to be the essence of Say's law (Say 1834) that eventually became imbedded in the supply side neoclassical growth models although it was repudiated in varying degrees in the writings of Ricardo, Malthus and Marx, who were concerned about the class structure and how it affected income distribution, supply, demand, and economic growth.

David Ricardo's limits on land productivity and population growth

David Ricardo was a contemporary of Malthus and a forerunner of Marx. He outlined the principles of distribution between the various economic classes, landlords, capitalists and workers, which later became important building blocks of the model of growth and decline of capitalism that Marx conceived. Last, but not least, he brought in the constraints to growth by stating his law of diminishing returns to land cultivation and the so-called iron law of wages (Ricardo 1817, McCulloch 1881). These constraints are added to the model of Figure 1 as follows:

a) Adding Ricardo's principle of diminishing marginal rents of land

Ricardo's definition of land rent equated it to land productivity. To quote Ricardo,

Rent is that portion of the produce of the earth which is paid to the landlord for the use of the original and indestructible powers of the soil. It is often however confounded with the interest and profit of capital…. (Ricardo 1817, ch. 2).

This means land will need to be disaggregated from capital in the production schedule, however, for simplification profits and rents can still be aggregated as the two are residuals after meeting wage bill and running expenses. According to Ricrado,

Whenever, then, the usual and ordinary rate of the profits of agricultural stock, and all the outgoings belonging to the cultivation of land, are together equal to the value of the whole produce, there can be no rent. And when the whole produce is only equal in value to the outgoings necessary to cultivation, there can neither be rent nor profit… (Ricardo 1815).

Adding a constraint driven by the land-capital ratio to the output in the model of Figure 1 creates diminishing marginal returns to land as conceived by Ricardo. Such a constraint would slow down the rate of growth of output, but would not bring it to a halt as long as the sum of marginal increases in output from additional investment into capital and the technological growth it creates outweigh the decrease in the marginal productivity of land. This means the relationship between investment and technological growth would be critical to maintaining growth in the face of diminishing land productivity. The resource constraint corresponding to Ricardo's principle of diminishing returns is added in Figure 4 while profits and rents are still aggregated.

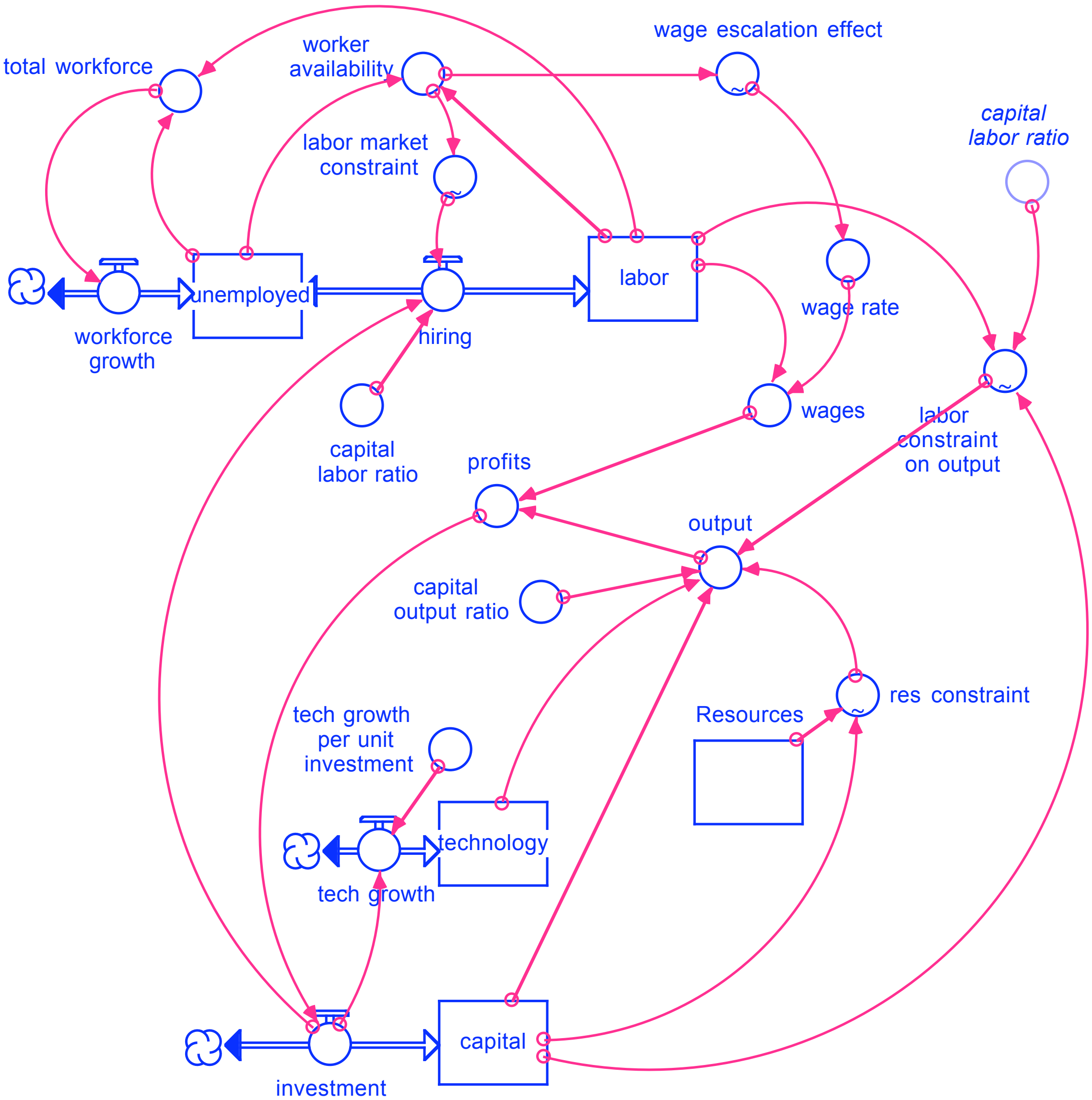


Figure 4 **Ricardo's law of diminishing land rents (productivity of renewable resources) added to the model.**

The production function determining output in the model of Figure 1 is modified as follows:

output	= tech*(capital/capital output ratio)*labor constraint on output*resource constraint
resource constraint	= f_4(resources/capital) ; $f_4' > 0$; $f_4'' < 0$
resources	= 200

The stock of resources remains constant in line with Ricardo's specification of "indestructible powers of the soil" - meaning that resources are fully renewable and thus do not deplete. The fixed value of resources is kept high with respect to capital to assure that the resource constraint is inactive at the start of the simulation. Figure 5 shows simulations of the output with different values of technological growth per unit of investment, shown as λ.

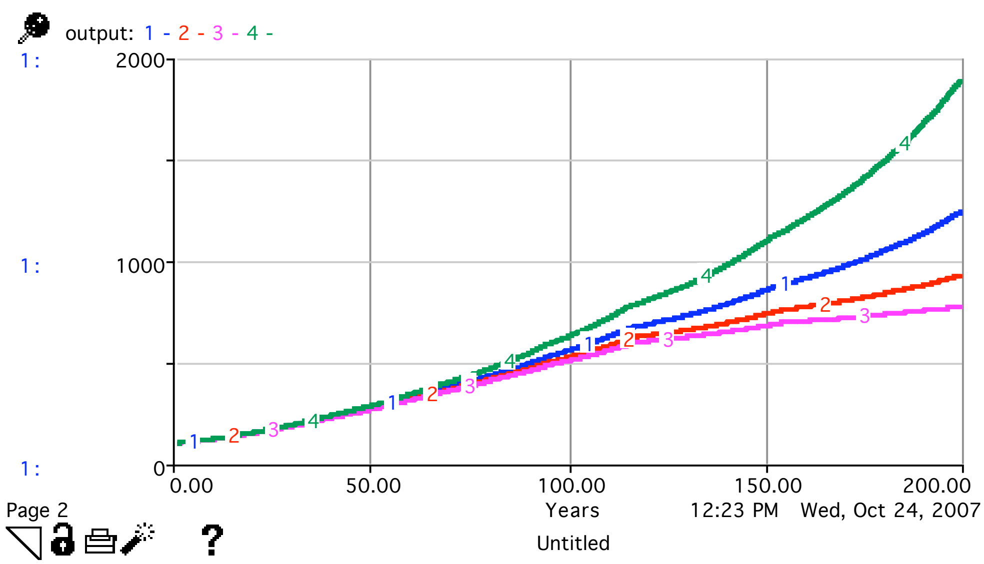


Figure 5 Effect of diminishing land productivity on output with different technological growth rates

The length of the simulation is increased to observe the effect of the newly introduced constraint. As expected, growth rate is slower than in Figure 3, while output moves to a new plateau when technological growth rate cannot offset the diminishing land productivity. However, as population continues to grow, the unemployed pool will continue to rise, which is anomalous since it would not be possible to feed an army of the unemployed so created. This anomaly is removed by adding the structure of Ricardo's iron law of wages to the model.

b) Adding Ricardo's iron law of wages

Ricardo's iron law of wages links population growth to the wage bill and predicts that population would grow until wage rates equilibrated at a subsistence level (Ricardo 1817, ch 5). The wage bill divided by subsistence wage, therefore, returns the demographic capacity to supply labor. When this law is implemented in the face of fixed land creating diminishing marginal returns to land, each additional unit of output would require more extensive use of capital and labor. However, as labor growth rate declines in response to a wage bill constrained by a diminishing wage rate and the population comes to a balance, the production reaches a plateau where the wage bill drives the profits to zero while the marginal product of labor nears subsistence wage.

The model I have presented in Figure 4 is modified further to incorporate the structure underlying the iron law of wages suggested by Ricardo. Figure 6 shows the model with this modification.

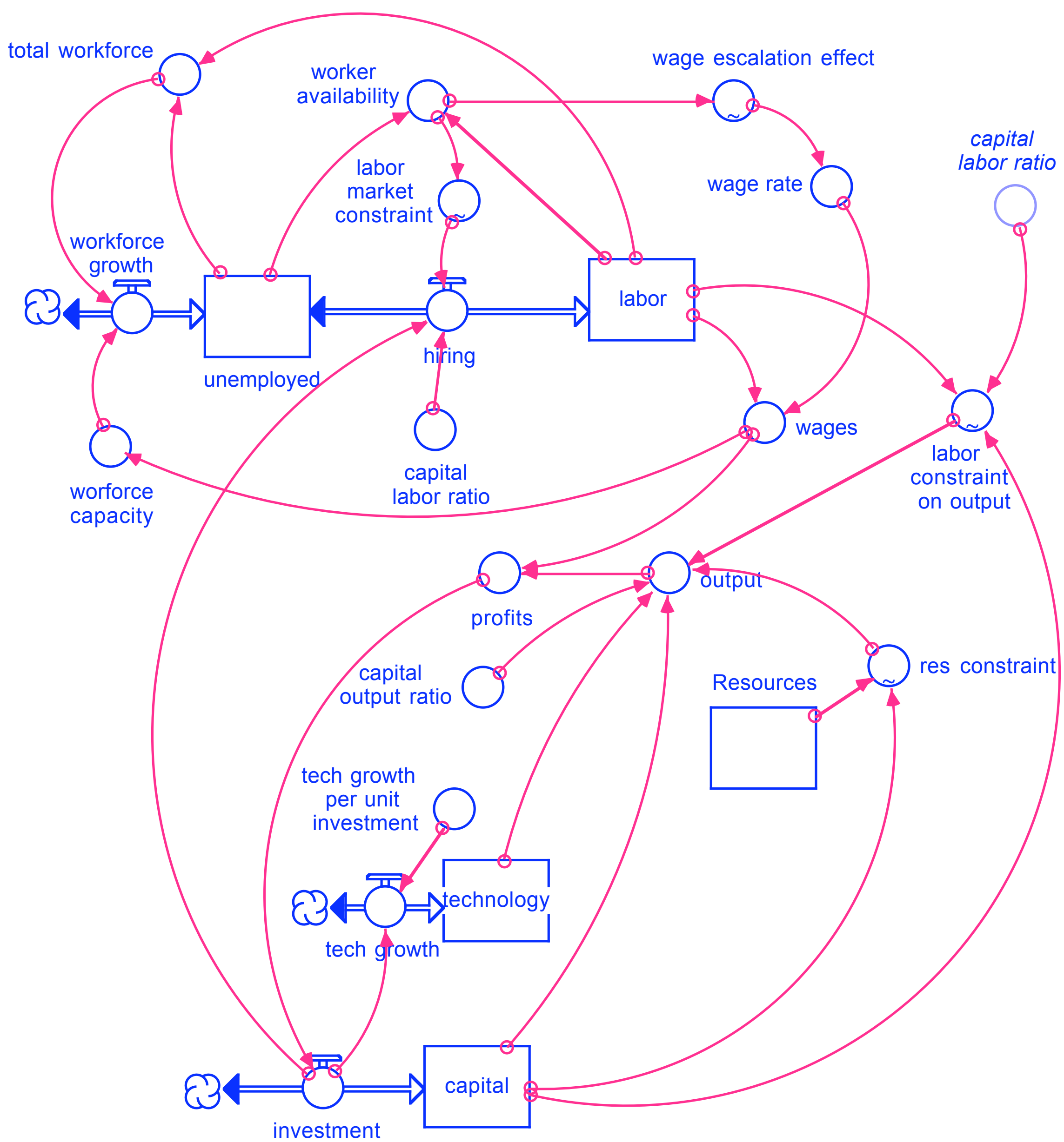


Figure 6 **Ricardo's iron law of wages and the concept of diminishing rents added to the model**

Wage bill divided by subsistence wage rate now determines the demographic capacity to supply labor. Workforce growth rate is driven by the discrepancy between the demographic capacity and the current workforce, while all growth in workforce feeds into the stock of unemployed from where labor is hired. Following additional mathematical relationships are created with this modification:

workforce growth	=(workforce capacity - total workforce)/ workforce adjustment time
workforce capacity	= wages/subsistence wage rate
workforce adjustment time	= 5
subsistence wage rate	= 6

Figure 7 shows the behavior of this modified model incorporating both Ricardo's principle of diminishing land rents and the iron law of wages. The wage rate rises at first and profits decline as the economy grows faster than the labor supply thus creating tightness in the labor market, but as marginal output declines while workforce continues to grow, a rising unemployment rate suppresses wage rate and it comes to a balance near the specified subsistence level. The profit (which subsumes land rents) grows after the initial dip caused by an increased wage bill, but it eventually decline to zero as the value of produce is all used up in paying the wage bill.

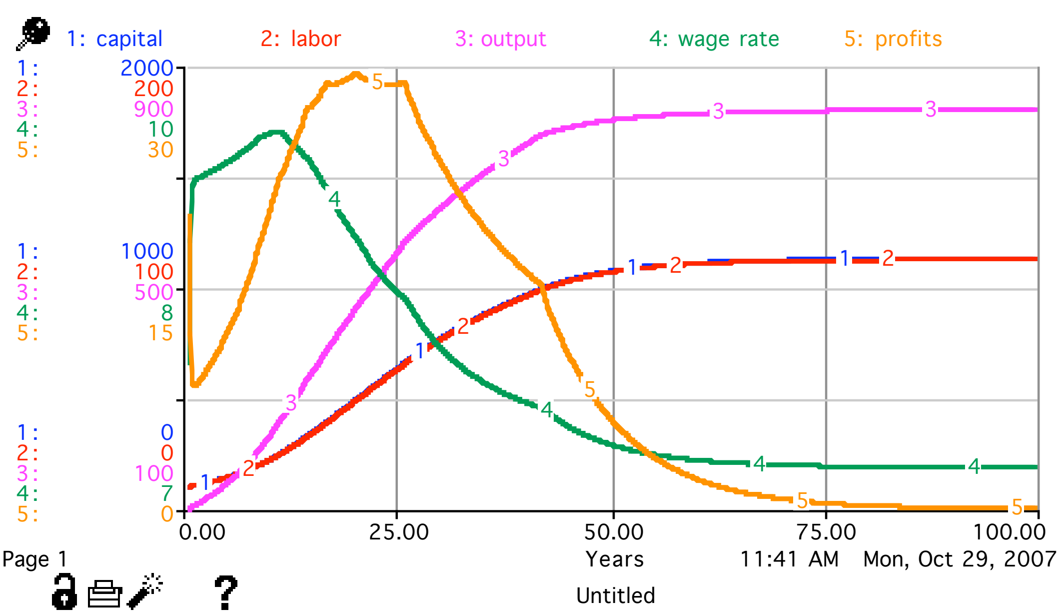


Figure 7 **Simulation of the Ricardian model of economic growth.**

In the final equilibrium, the wage rate equilibrates at near subsistence level, while profits decline to zero.[2] The population has grown to the level determined by the wage bill that provides enough subsistence to the workers so they can produce, but no not enough for procreation. Please note that population growth depends on the wage bill only and not on the total output, which implies that profits are not received by the working households while capitalist households continue to invest profits until they decline to zero irrespective of the rate of return on capital. Ricardo did distinguish between "natural" and "market" prices of commodities meaning that he was are of the imbalance between supply and demand (Ricardo 1817, ch 4). However, he did not openly repudiate Say's law as the production resources in his model always remain fully employed and the markets apparently clear.

Thomas Malthus, Jay Forrester, population growth and depletion of resources

Thomas Malthus, published ideas similar to Ricardo's almost simultaneously as Ricardo. He surmised that population growth by itself is not enough to bring economic advances. He felt that population growth is an *end product* in the economic growth process, rather than a means and posited that an increase in population cannot take place without a proportionate or nearly proportionate increase of wealth (Malthus 1798). Malthus repudiated Say's law by differentiating between the recipients of profits and wages and emphasizing the importance of demand that is linked mainly to the wage income, but he did not connect this factor to the demise of capitalism as Marx later posited (Higgins 1968, pp 67-75). Malthus later became concerned with what he described as population explosion and the scarcity of resources resulting from it (Malthus 1821), and expressed more or less similar ideas about procreation as Ricardo. The feedback relationship between population growth and economic growth is however more explicitly addressed

[2] There appears what is called in control jargon as steady state error between the nominal and actual values of subsistence wage in my model since the instantaneous rather than the cumulative discrepancy between workforce capacity and population drives population growth. If the discrepancy were integrated in a stock which is then used to drive population growth, the difference between the nominal and steady state values of subsistence wage would disappear, but the growth process would become unstable. This process is called integral control, which is used in Phillip's stabilization policy (Phillips 1854), but is outside of the scope of the model of this paper.

by Ricardo through his iron law of wages and the principle of diminishing marginal rent to land as represented in Figure 6 than by Malthus in his essay on population (Malthus 1798) and in his Principles of Political Economy (Malthus 1921).

It is also not clear whether Malthus considered resources in the framework of fixed land, which does not get depleted or nonrenewable resources, which get depleted. Hypothetically, if a resource depletion process is added to the Ricardian model of Figure 6, an overshoot and decline behavior outlined in Forrester's World Dynamics and the Limits to Growth/Beyond the Limits studies is obtained (Forrester 1971, Meadows et. al., 1972, 1974, 1992). The structural modifications needed for this are shown in Figure 8. They add the following depletion relationships to the model:

resources = ∫(-resource depletion)dt
resource depletion = output*resources used per unit of output
resources used per unit of output = .02

Figure 9 shows the behavior of the modified model of Figure 8, which is similar to the overshot and decline behavior postulated by Forrester. Forrester has sometimes been accused of replicating the Ricardian/Malthusian model, but he clearly has dealt with nonrenewable resources while the earlier thinkers seemed to be dealing with non-depleting land or renewable resources. Also Forrester disaggregated the limits into an array that further dealt with food shortage and environmental degradation arising out of economic growth and population growth, which could create constraints on growth while material resources were still plentiful. He also introduced the concept of decisions in bounded rationality and the delays in recognition of the information on which the bounded rational decisions of economic actors are based and how these limits could cause an overshoot and decline in population (Radzicki 1988, Morecroft 1985). This way, Forrester provided a far more succinct theory of limits to growth than posited in the classical economic theories. In terms of the holistic nature of his theory and the magnificent dynamics it created, Forrester's model indeed should be placed with the classical models of models of economic growth.

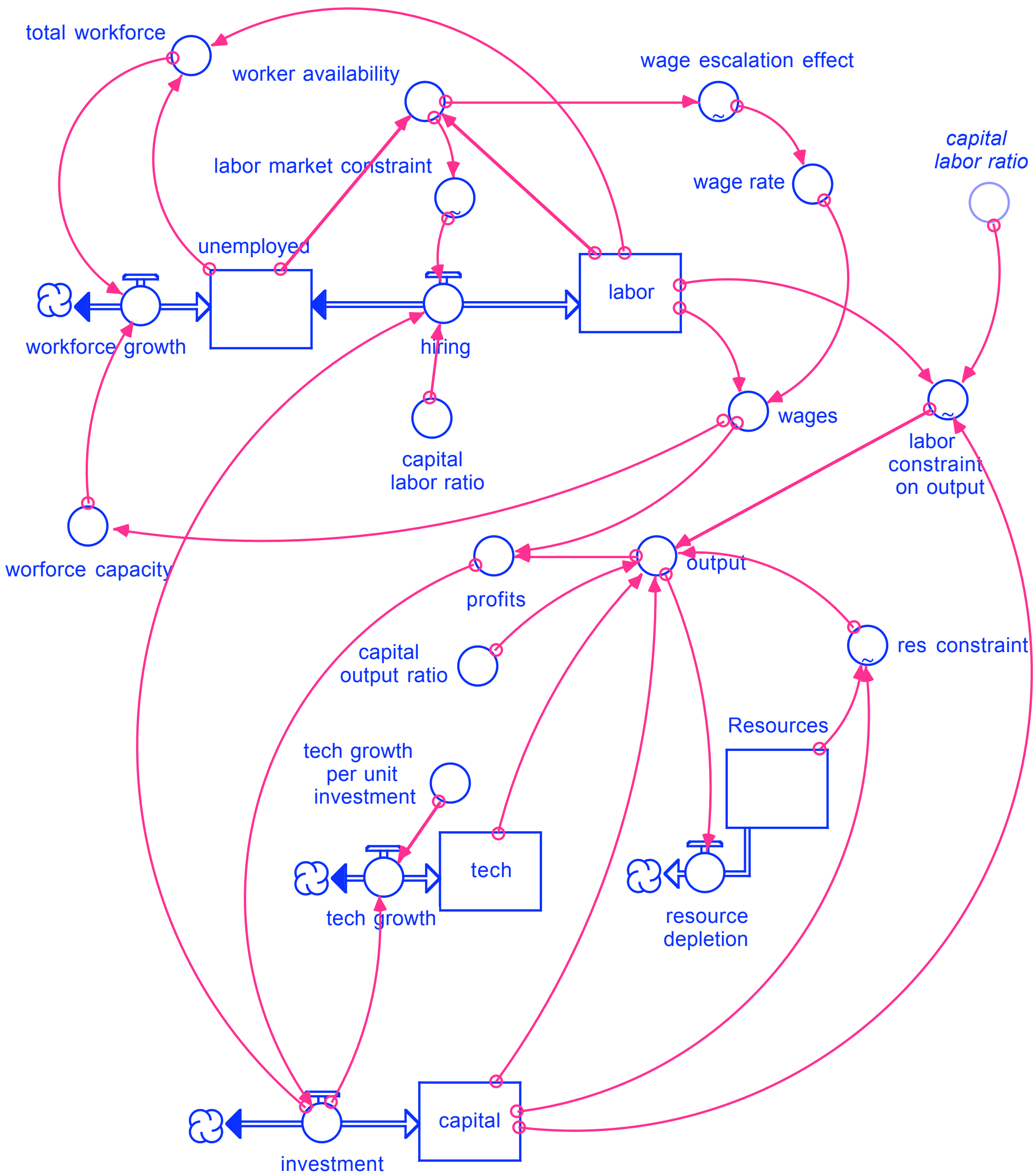


Figure 8 **Ricardian/Malthusian model with depleting resources**

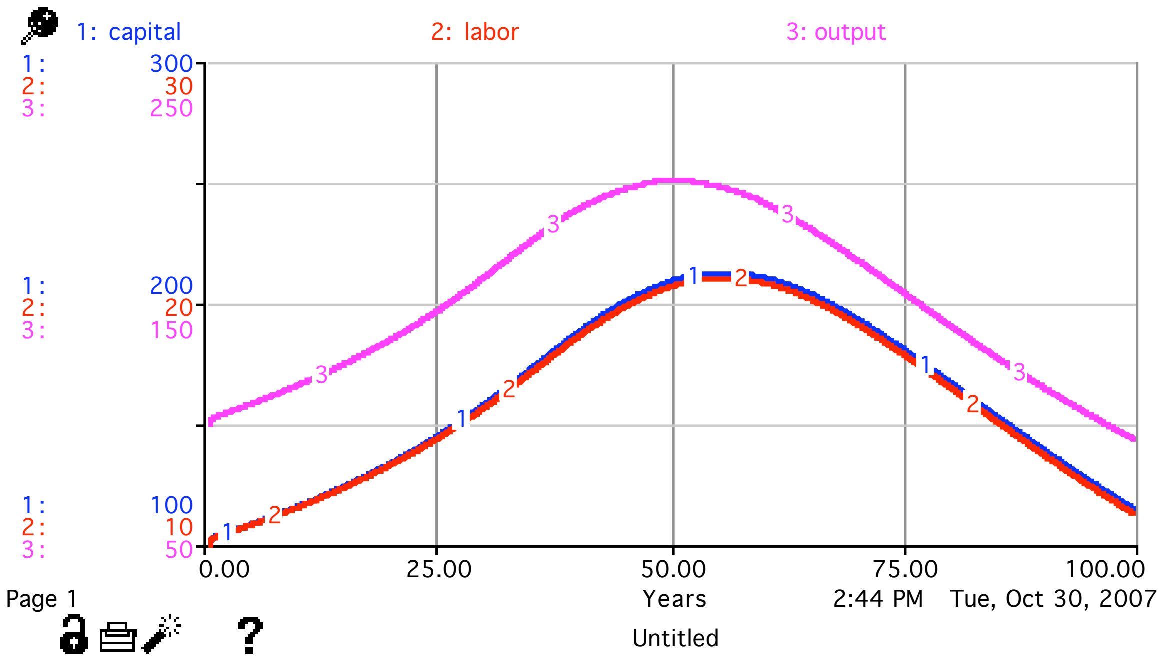


Figure 9 **Overshoot and decline behavior obtained from Ricardo's model with depleting resources**

Marx's model of the downfall of capitalism

Marx in his monumental work, Capital, added a new dimension to the concept of limits to growth by tying them to the social and political factors (Marx 1906). He saw these limits arising out of social conflict emerging from income distribution rather than resource limitations. He took an exploitative view of economic growth and posited that it arose out of appropriation of the surplus value by the capitalists. Such exploitation is however made possible only when there is a large pool of unemployed labor so workers can bargain for only subsistence wage irrespective of their contribution to production, which is achieved by the capitalists by creating labor-substituting technological advances (Higgins 1968).

Marx distinguished between the use value and exchange value of a commodity, the later being proxied by the market price. He also postulated a social division of labor, in which different people produced different products, so an exchange could occur. As the ultimate volume of demand for these commodities emerged from the disposable income of the households, a large pool of unemployed would eventually stifle this demand. Marx thus clearly repudiated Say's Law.

Marx also introduced the concept of rate of return on capital which was affected by the exchange value of commodities. The rate of return influenced the rate of investment. Marx's logic is sometimes criticized since in his model investment continues even when the rate of return turns down. He assumed that available profit will be invested until the rate of return goes to zero, while profit is the result of the labor performed by the worker beyond that necessary to create the value of her wages. Thus profit arose out of the surplus value of labor, which is referred to as the surplus value theory of profit.

This investment structure was, however, consistent with Marx's distinction between the capitalists who received all profits and did not have to accrue any capital costs to justify an investment decision, and the asset-less proletariat who received only wages. Thus, unlike the neo-classical model, the rate of return in Marx's model was not the only factor determining investment. So, even when the rate of return declined, surplus value accrued as profits needed to be invested. Only when both profits and the rate of return became zero did the investment finally atrophy. Marx did indeed make the prediction that the rate of profit will fall over time, and this was one of the factors which led to the downfall of capitalism. The rate of return declines as the unemployed proletariat is unable to buy the end commodities and the production capacity cannot be utilized, leading to the creation of idle capital (Wolff 2003, Higgins 1968, pp 76-87).

Figure 10 shows the essential structure of Marx's model that is common to the earlier models of this paper with the difference that technological development is assumed to be labor substituting. Output is expressed in terms of use value and even though price is computed for determining the rate of return on investment, exchange value is not explicitly represented. Table 3 gives the mathematical relationships underlying the model structure in Figure 10.

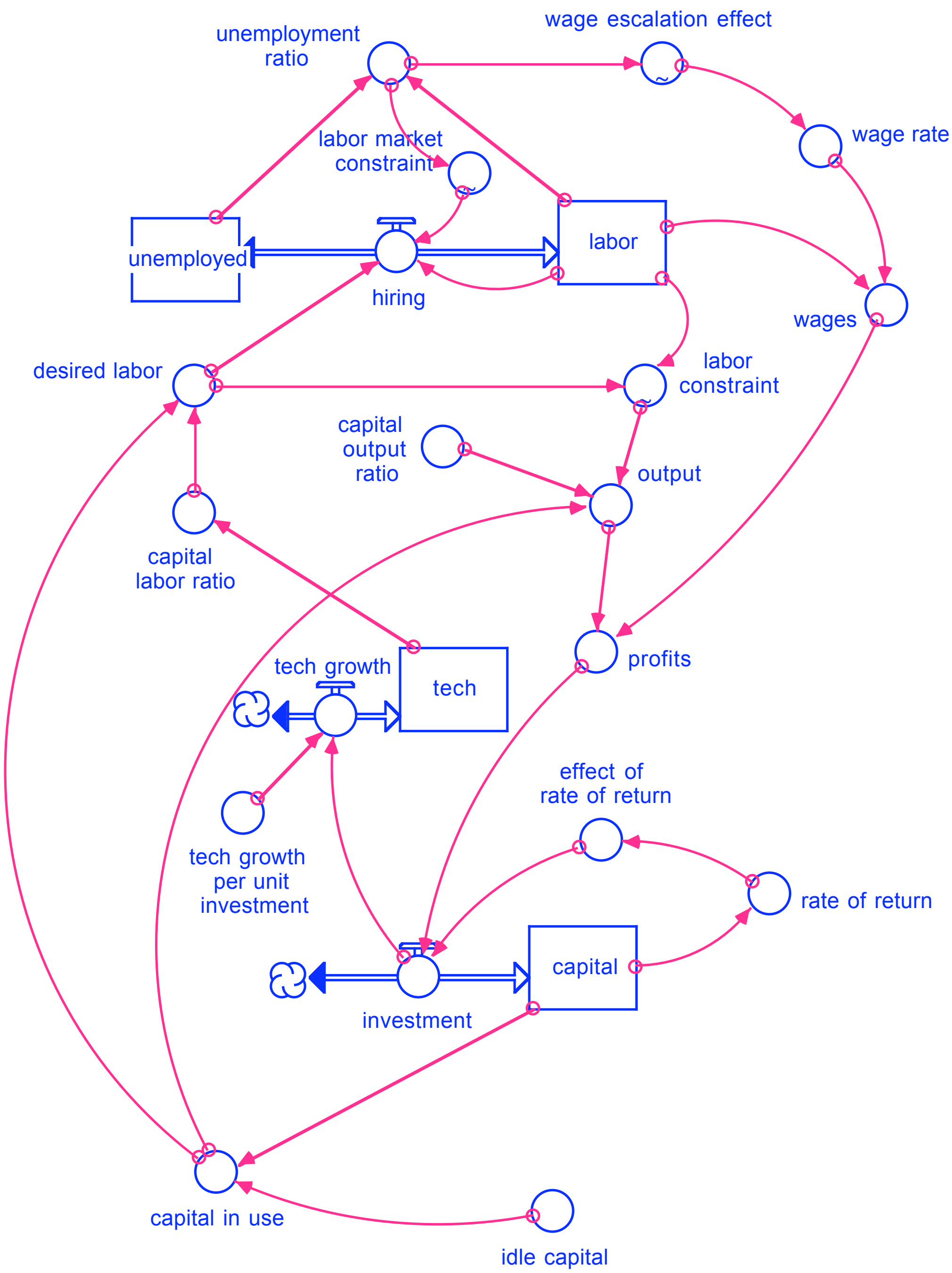


Figure 10 **Labor substituting technological development, rate of return and idle capital added to the growth process as conceived by Marx**

Table 3 Mathematical relationships corresponding to the partial structure of Marx's model in Figure 10

VARIABLES	CONPUTATIONAL EQUIVALENTS
a) Stocks and Flows	
capital	∫(investment)dt
investment	profits*effect of rate of return
labor	∫(hiring)dt
unemployed	∫(-hiring)dt
hiring	((desired labor-labor)/labor adjustment time)*labor market constraint
tech	∫(tech growth)dt
tech growth	investment*tech growth per unit investment
b) Converters	
capital in use	capital-idle capital
capital labor ratio	normal capital labor ratio*tech
desired labor	capital in use/capital labor ratio
effect of rate of return	rate of return/normal rate of return
output	(capital in use/capital output ratio)*labor constraint
profits	output-wages
rate of return	profits*price level/capital
unemployment ratio	(unemployed/labor)/(INIT unemployed/INIT labor)
wages	labor*wage rate
wage rate	normal wage*wage escalation effect
c) Graphical functions	
labor constraint	f_1(labor/desired labor); $f_1' > 0$, $f_1'' < 0$
labor market constraint	f_2(worker availability); $f_2' > 0$, $f_2'' < 0$
wage escalation effect	f_3(worker availability); $f_3' < 0$, $f_3'' < 0$
d) Initial values of stocks	
INIT capital	100
INIT labor	10
INIT tech	1
INIT unemployed	2
e) Constant parameters	
capital output ratio	1
tech growth per unit investment	0.01
normal rate of return	0.2
normal wage	8
normal capital labor ratio	10

Note that while production and labor market relationships are similar to the initial model of Figure 1 and profit is still calculated as a residual quantity, technology now affects capital labor ratio rather than the output. Also, the rate of return affects the investment decision in addition to the profits and the capital is divided into two categories, capital in use and idle capital. The hiring depends on the discrepancy between desired labor and labor instead of being directly driven by the investment rate. The desired labor in turn is determined by the capital in use and the capital labor ratio. Figure 11 shows the complete model. The rate of return on capital is determined by the use value of the commodities constituting profit per unit of capital multiplied by price.

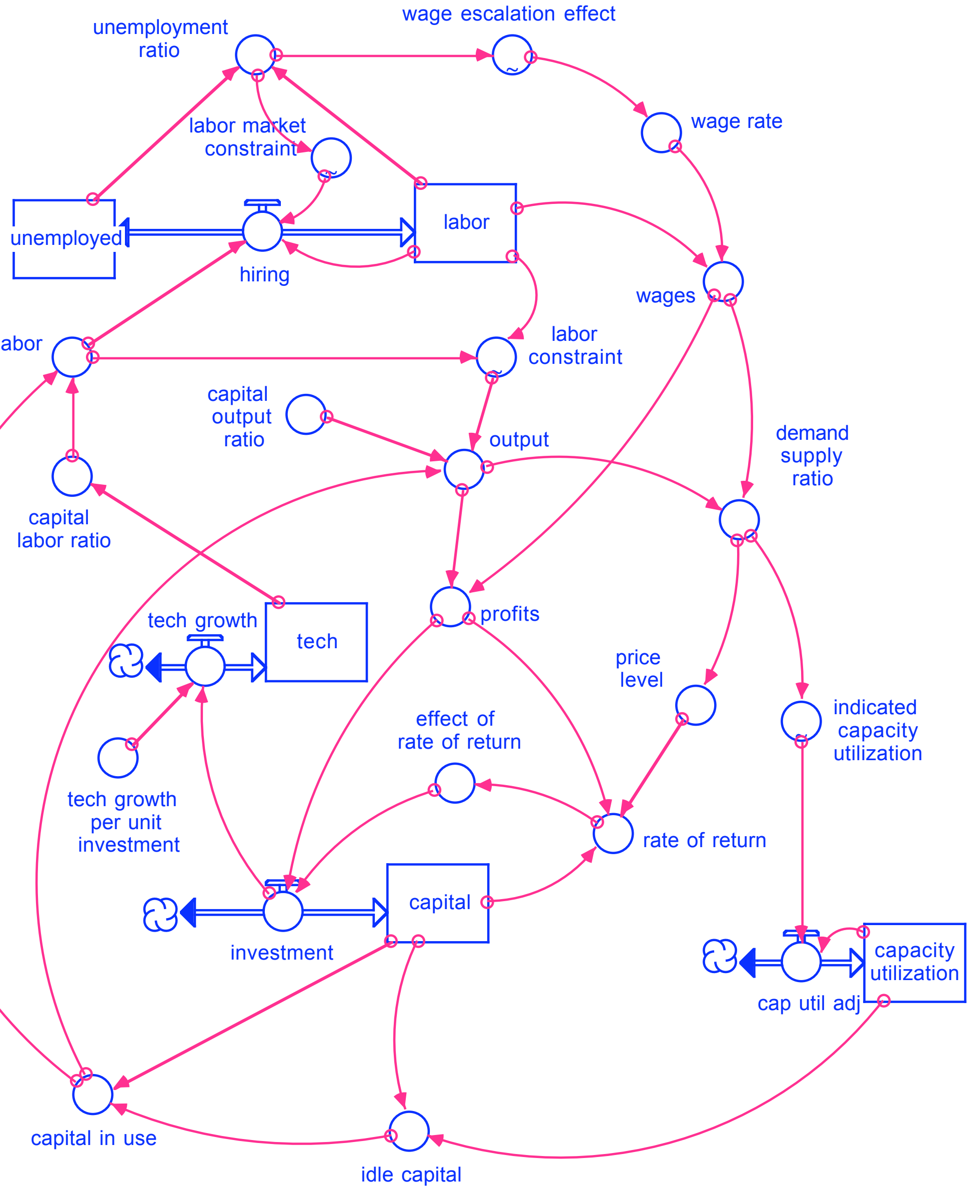


Figure 11 **Marxian model of economic growth**

The price in turn depends on supply and demand. Here is where Say's Law is fully repudiated. The demand depends on the wage bill while the supply is created by the capital in use and the employed labor. The capital in use is the difference between the capital and the idle capital, which depends on capacity utilization. Capacity utilization, in turn, is determined by the demand relative to the supply over the past period. Population growth rate is assumed to be zero, while constraints arising from limited resources as suggested by Ricardo and Marx are excluded. Table 4 gives the additional mathematical relationships representing above logic. Figure 12 shows the simulated behavior of this growth model.

Table 4 Additional mathematical relationships added to complete the Marxian model in Figure 11

VARIABLES	COMPUTATIONAL EQUIVALENTS
a) Stocks and Flows	
capacity utilization	∫(capacity utilization adjustment)dt
capacity utilization adjustment	(indicated capacity utilization - capacity utilization)/capacity utilization adjustment time
b) Converters	
demand supply ratio	wages/output
price level	demand supply ratio/market clearing demand supply ratio
c) Graphical functions	
indicated capacity utilization	f_4(demand supply ratio/.8); $f_4' > 1$, $f_4'' < 1$
d) Initial value of stocks	
INIT capacity utilization	1
e) Constant parameters	
market clearing demand supply ratio	0.8
capacity utilization adjustment time	4

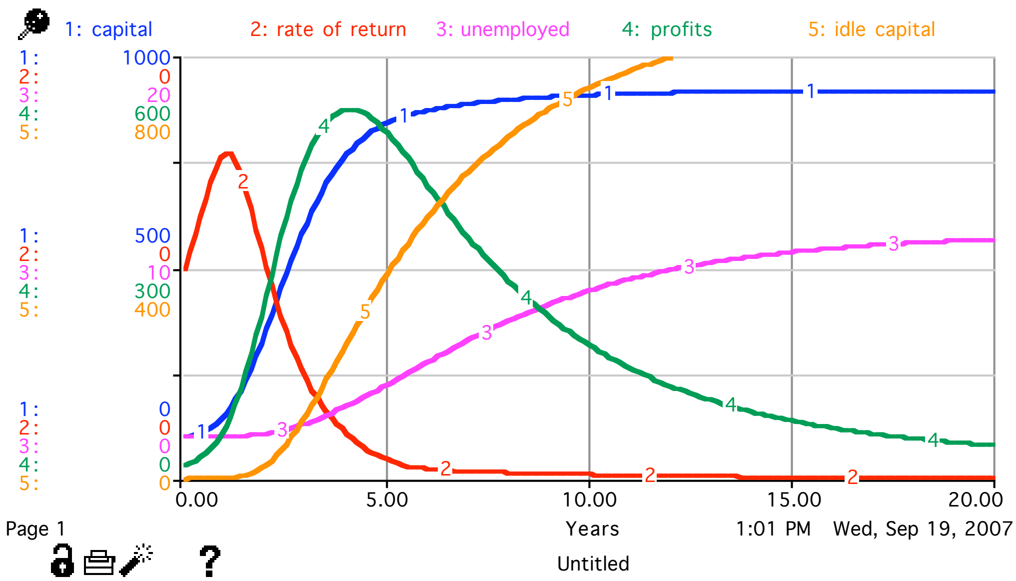


Figure 12 **Decline of rate of return and profits, and the creation of a reserve army of the unemployed in the simulation of Marx's model of economic growth**

As postulated by Marx, the relationships in his model do indeed lead to a growth and collapse behavior in the rate of return and profits as capital grows along with a reserve army of the unemployed since new investments are labor substituting. Investment is driven down to zero when both the rate of return and the rate of profit go to zero. Meanwhile, the capacity utilization shrinks and idle capital stock rises.

The decline in profits is due to the growth in idle capital rather than the wage bill since the reserve army of the unemployed keeps wage rate at subsistence level. This can be a conflictful scenario that Marx suggested signaled the end of capitalism. It is not clear whether Marx thought the reserve army of the unemployed would destroy idle capital (Baumol 1959), although he postulated that the uprising of the masses would be concomitant with such destruction. Either way, the stock of physical capital would decay as suggested by the additional structure in Figure 13, adding the following equations to the model:

capital	$= \int(\text{investment} - \text{capital decay})dt$
capital decay	= idle capital/decay time
decay time	= 20*effect of destructive forces
effect of destructive forces	$= f_5(\text{unemployment ratio});\ f_5' < 1,\ f_5'' < 1$

The simulated behavior arising from this structure with and without the effect of the destructive forces is shown in Figures 14 and 15. The decay is faster when the destructive forces arising from the reserve army of the unemployed are taken into account and slower without them, but the trend is the same in both cases.

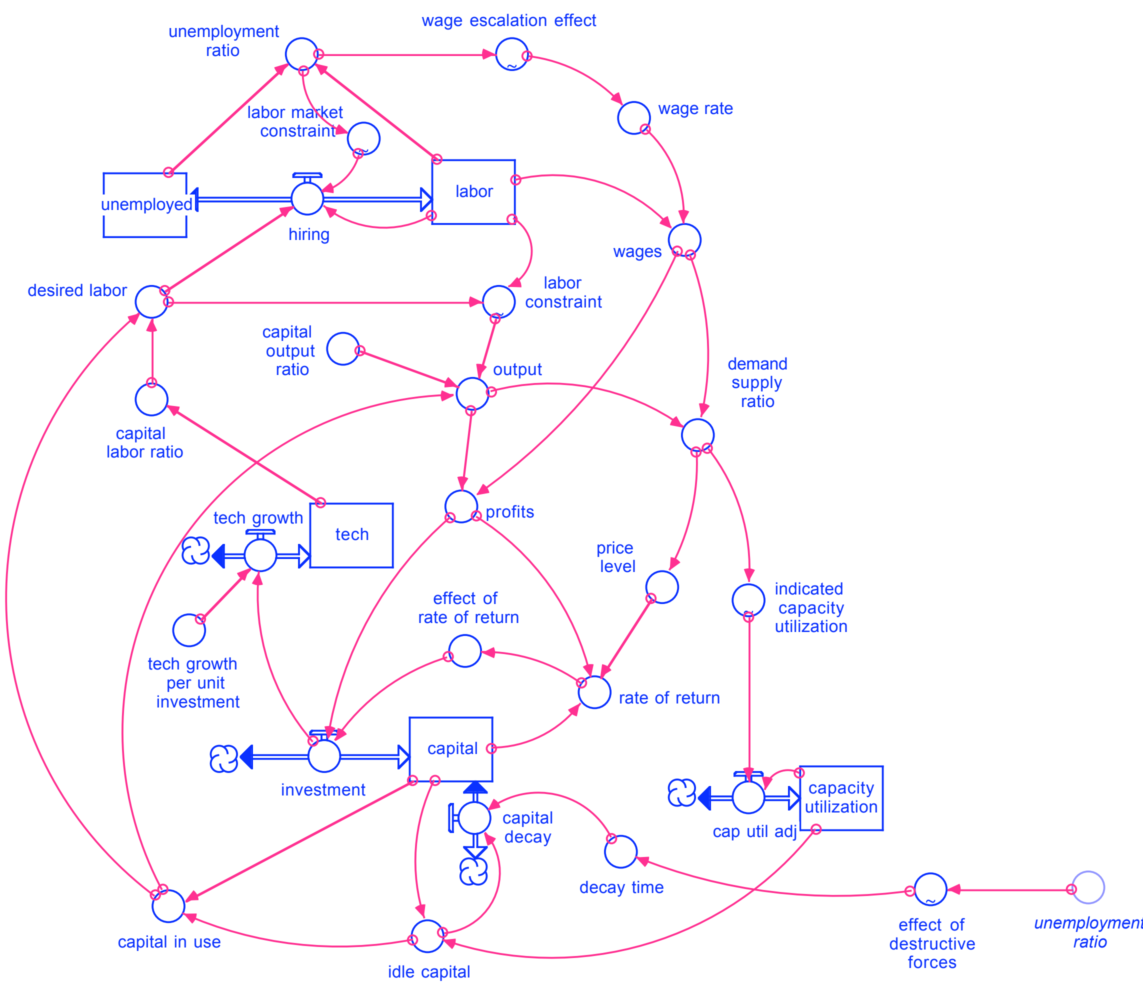


Figure 13 Capital decay added to Marxian model

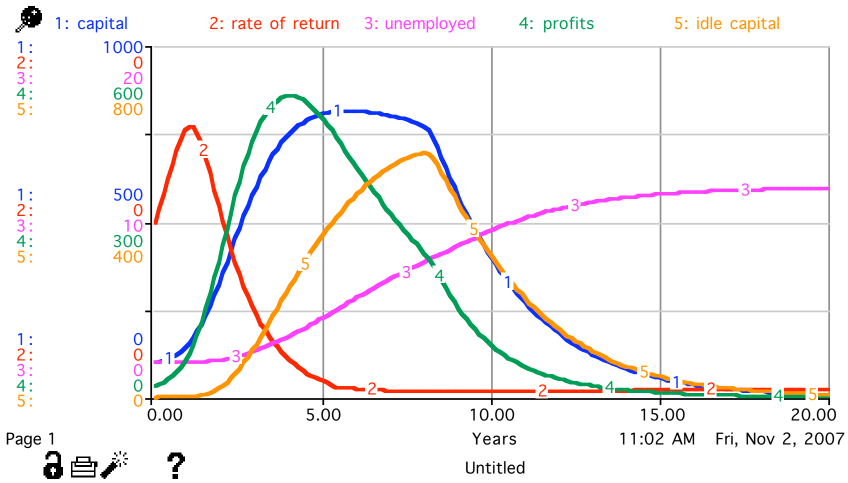


Figure 14 Decay of capital with a disruptive reserve army of the unemployed

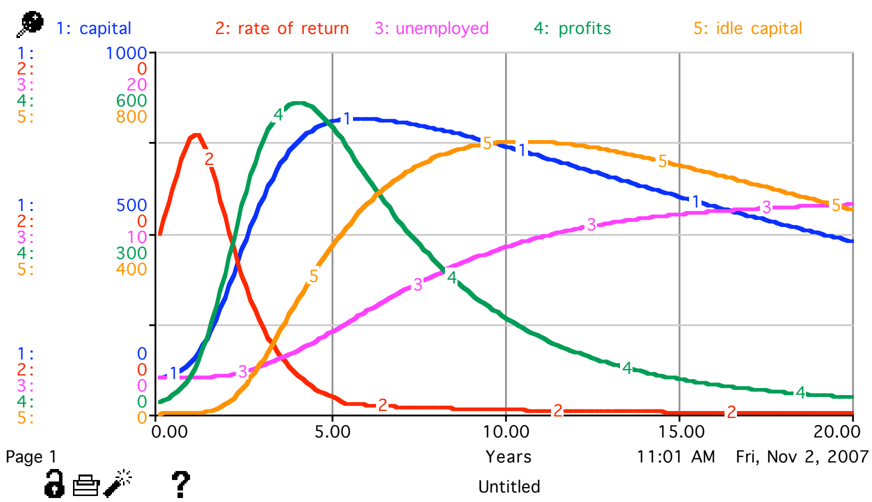


Figure 15 Decay of capital with a peaceful reserve army of the unemployed

Schumpeter's concept of creative destruction and economic cycles

While Marx's model of destruction of capitalism through exploitation of the proletariat was based on a class system that locked capitalists and proletariat in separate compartments, Schumpeter saw the possibility that entrepreneurship could exist across all social classes. Thus new entrepreneurs could emerge from the ruins of a fallen capitalist system. They could create a resurgence of capitalism from an environment in which cheap labor and the possibility of profiting from it would allow them to mobilize idle capital resources and create new and marketable goods and services from them. In my observation, Schumpeter saw the possibility of social mobility between classes arising from entrepreneurship that would rejuvenate a declining capitalist economy, while Marx had ruled out such mobility. Schumpeter pointed out that entrepreneurs innovate, not just by figuring out how to use inventions, but also by introducing new means of production, new products, and new forms of organization. These innovations, he argued, take just as much skill and daring as does the process of invention (Schumpeter 1962).

Figure 16 shows the production system and labor market structure implicit in Schumpeter's descriptive model as outlined by Higgins (Higgins 1968, pp 88-105). Please note this structure is more or less similar to Marx with the exception that labor-substituting characteristic of technology is omitted and the direct link between profits and investment is deleted. Schumpeter, in fact, distinguished between two types of investment that he called *induced* and *autonomous*. He also introduced a concept of "*saving up*" which is different from saving in the neoclassical growth model. Saving up constituted the part of output that is withheld from investment and consumption. Induced investment arose from the discrepancy between supply and demand and autonomous investment from resources and technology created by the entrepreneurs. Table 5 gives the mathematical relationships underlying the partial structure of Schumpeter's model outlined in Figure 15.

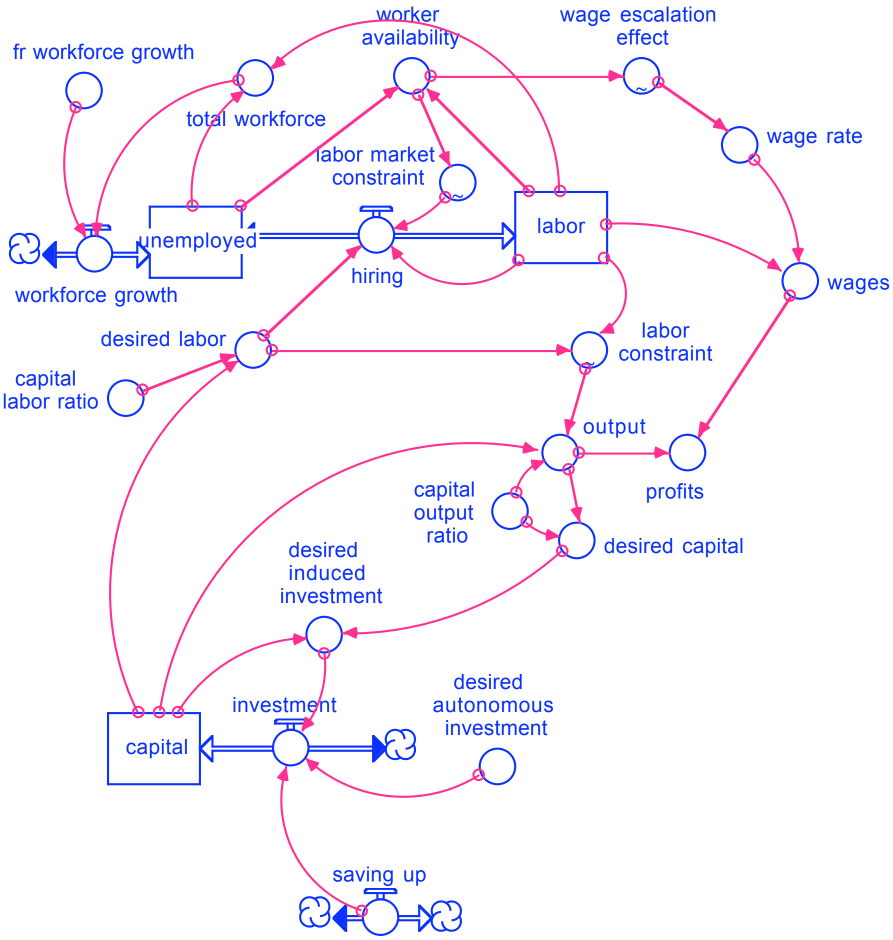


Figure 16 **The production system and the labor market implicit in Schumpeter's model.**

Table 5 Mathematical relationships corresponding to the partial structure of Schumpeter's model in Figure 16

VARIABLES	COMPUTATIONAL EQUIVALENTS
a) Stocks and Flows	
capital	∫(investment)dt
investment	desired induced investment + desired autonomous investment-saving up
labor	∫(hiring)dt
hiring	((desired labor-labor)/2)*labor market constraint
unemployed	∫(workforce growth – hiring)dt
Workforce growth	total workforce*fractional workforce growth
b) Converters	
desired capital	output*capital output ratio
desired labor	capital/capital labor ratio
desired induced investment	(desired capital-capital)/5
output	(capital/capital output ratio)*labor constraint
profits	output-wages
total workforce	labor + unemployed
wages	labor*wage rate
wage rate	8*wage escalation effect
worker availability	(unemployed/labor)/(INIT unemployed/INIT labor)
c) Graphical functions	
labor constraint	f_1(labor/desired labor); $f_1' > 0$, $f_1'' < 0$
labor market constraint	f_2(worker availability); $f_2' > 0$, $f_2'' < 0$
wage escalation effect	f_3(worker availability); $f_3' < 0$, $f_3'' < 0$
d) Initial values of stocks	
Initial values of stocks	
INIT capital	100
INIT labor	10
INIT unemployed	2
e) Constant parameters	
capital labor ratio	10
capital output ratio	1
fractional workforce growth	0

Saving up, possibly extended across social classes and fueled entrepreneurial activity leading to autonomous investment. Although one does not get a clear sense of this process from the descriptive writings of Schumpeter, I detect recognition of social mobility in this concept that allows workers to become the new capitalists. In the complete model shown in Figure 17, I would make a small amendment to Schumpeter's concept of entrepreneurs creating resources; I would call it mobilizing resources accumulated through saving up, mainly to designate a source of these resources in a formal model.

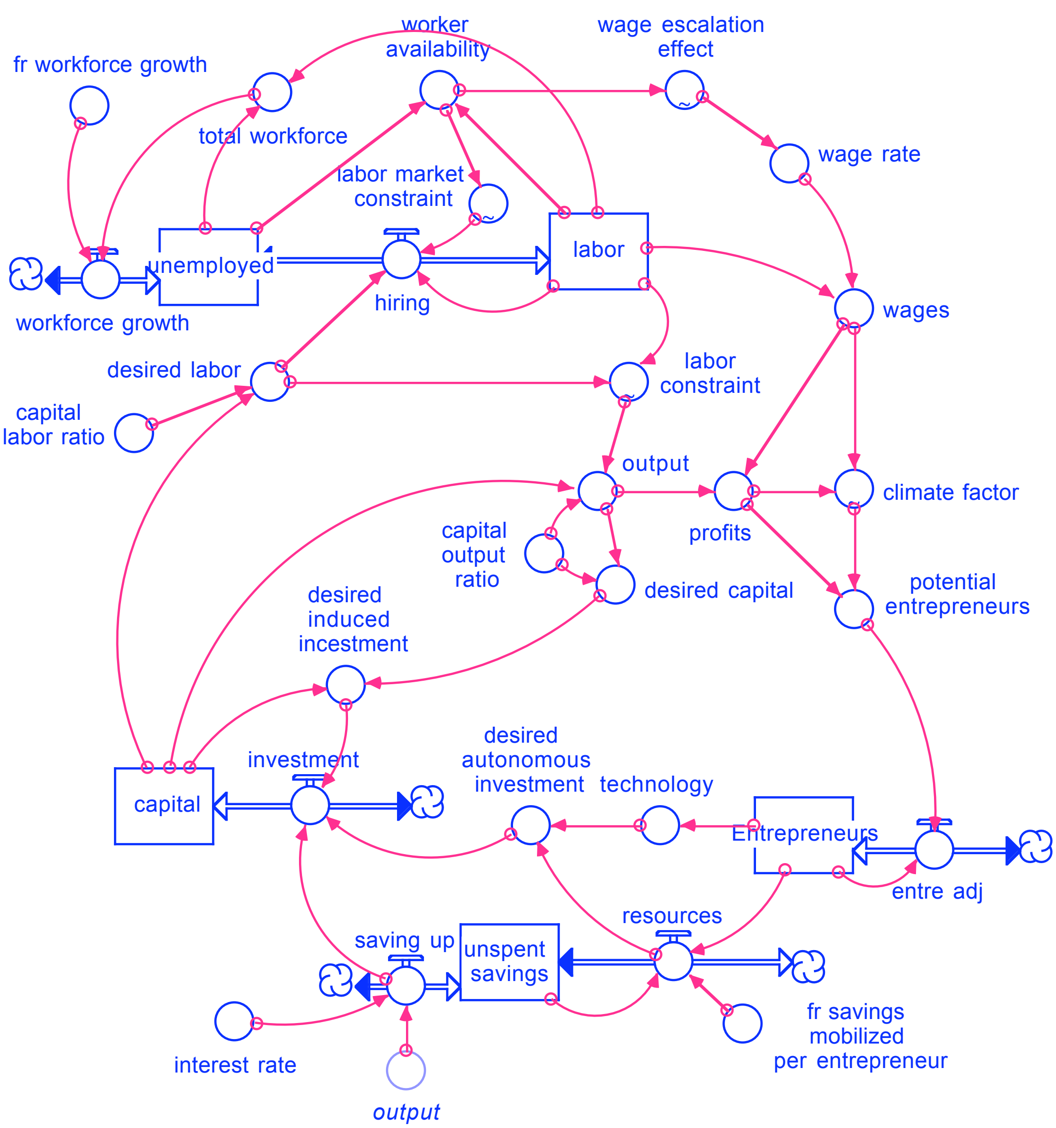


Figure 17 Complete structure of Schumpeter's model of creative destruction

Both mobilized resources and technology depend on the number of entrepreneurs, which adjusts towards their potential number determined by profits and entrepreneurial climate. According to Schumpeter, entrepreneurial climate is created by the availability of a high rate of profits and the availability of cheap labor. I have accumulated the difference between the saving up, which Schumpeter said depended on interest rate, and the mobilized resources in a stock of unspent savings, which supply the venture capital for the entrepreneurs. This also allows the model to have a hypothetical equilibrium in which induced investment is zero and saving up equals the resources mobilized by the entrepreneurs or the venture capital investment. Table 6 shows additional mathematical relationships corresponding to the additional structure in Figure 17.

Table 6 Additional mathematical relationships corresponding additional structure in Figure 17.

VARIABLES	COMPUTATIONAL EQUIVALENTS
a) Stocks and Flows	
unspent savings	∫(saving up – resources)dt
saving up	output*f_6[interest rate] ; f_6 > 0
resources	Entrepreneurs*fraction savings mobilized per entrepreneur*unspent savings
technology	∫(Entrepreneurs*technological productivity of entrepreneurs-technology)/technology adjustment time
Entrepreneurs	∫(entrepreneur adjustment)dt
Entrepreneur adjustment	(potential entrepreneurs-Entrepreneurs)/Entrepreneur adjustment time
b) Converters	
potential entrepreneurs	(profits*potential entrepreneurs per unit profit)*climate factor
desired autonomous investment	resources^.5*technology^.5
c) Graphical functions	
climate factor	f_7[profits/(profits + wages)/.2]; f_7'> 0, f_7" < 0
d) Initial values of stocks	
INIT Entrepreneurs	10
INIT unspent savings	20
e) Constant parameters	
Interest rate	0.1
fraction savings mobilized per entrepreneur	0.05
Technology adjustment time	5
Entrepreneur adjustment time	5
technological productivity of entrepreneurs	0.5
potential entrepreneurs per unit profit	0.5

Figure 18 shows the behavior of Schumpeter's model with a fixed labor supply. Figure 19 shows the behavior with an autonomous rate of growth in labor. The model shows the cycles extensively discussed by Schumpeter, although the variety of periodicities he referred to is not addressed in this model.

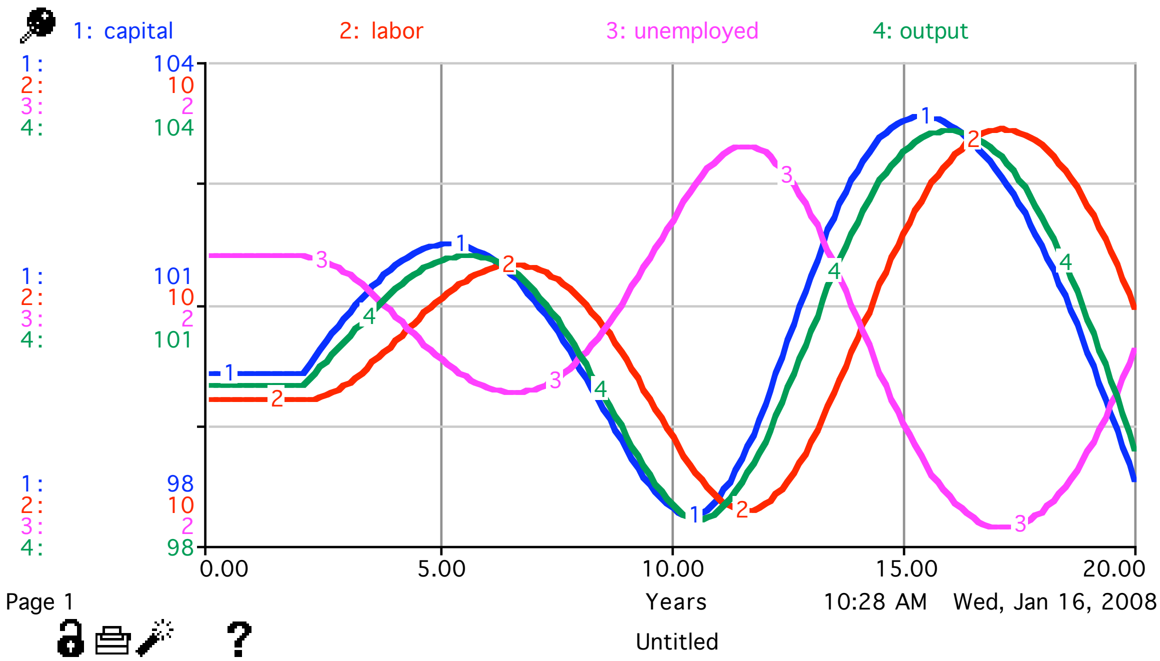


Figure 18 Behavior of Schumpeter's model without autonomous population growth

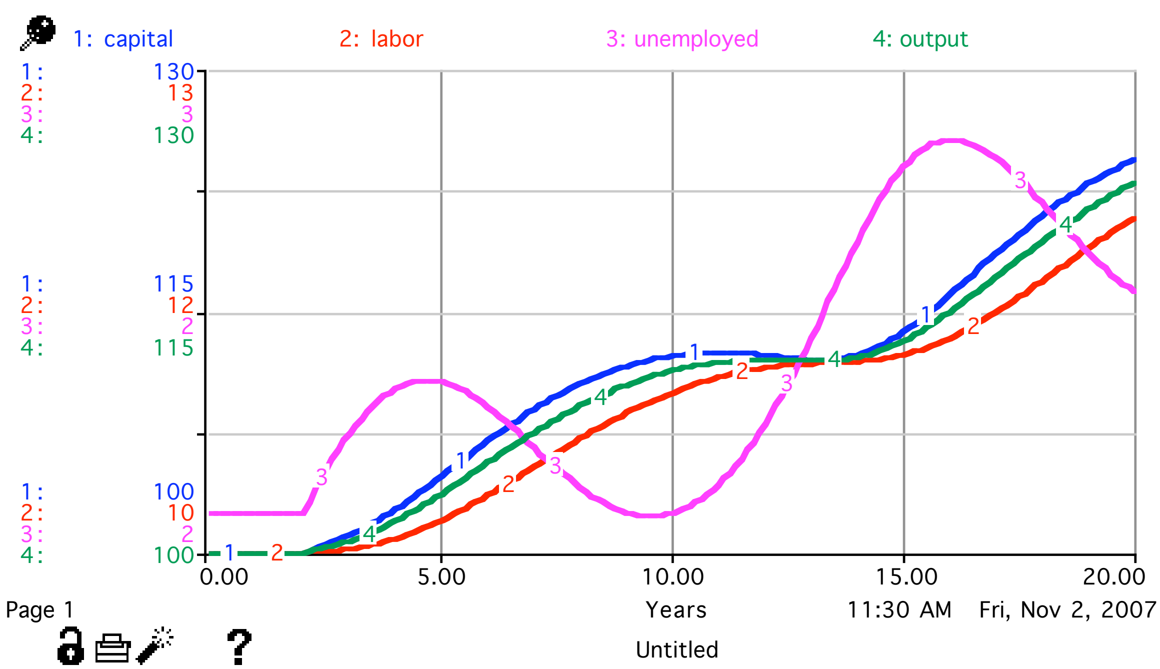


Figure 19 Behavior of Schumpeter's model with autonomous population growth

The autonomous investment arising from entrepreneurial creativity creates competition that expands creative activity and shrinks profits, while creating tightness in the labor market that takes away the very elements of the entrepreneurial environment that helped launch it. Schumpeter called this process the "creative destruction" and postulated that this would result in a cyclical tendency in the capitalist system, which is indeed borne out buy the simulation of his model. Although Schumpeter referred to many types of economic cycles in his writings the feedback processes distinguishing their periodicities are not clear. The model I have constructed exhibits a periodicity of about 10 years based on the time constants I have selected, while it specifically addresses the process of creative destruction that Schumpeter originally posited.

Conclusion

The concept of limits was tightly interwoven with the process of growth postulated in the classical theories. These limits encompassed many domains including demographic, environmental, social and political. In most instances, the recognition of these limits required dealing with soft variables that were difficult to quantify in the neoclassical analysis tradition. It is not surprising that these processes have been excluded from the formal analyses of mainstream economics, which has greatly reduced the explanatory power of the neoclassical theory, which has come to attribute all deviations from the postulated behavior of a hypothetical perfect market system to the imperfections in the reality, which is a violation of the scientific principles of modeling. All models are wrong, only reality is right and first requirement of a model is to replicate some aspect of reality before it can be accepted as a basis for a policy intervention.

Classical economics, on the other hand did attempt to replicate empirical realism in its theories often using soft variables in its explications. System dynamics modeling allows reinstatement of such soft variables in our models of economic behavior that should reincarnate the rich insights the traditional economic concepts provided. This indeed requires reinventing modern economics, which should be undertaken without further delay.

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